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Models, statistics, and rates of binary correlated sources



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ABSTRACT

This paper discusses and analyzes various models of binary correlated sources, which may be relevant in several distributed communication scenarios. These models are statistically characterized in terms of joint Probability Mass Function (PMF) and covariance. Closed-form expressions for the joint entropy of the sources are also overviewed. The asymptotic entropy rate for very large number of sources is shown to converge to a common limit for all the considered models. This fact generalizes recent results on the information-theoretic performance limit of communication schemes which exploit the correlation among sources at the receiver.

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1. Introduction

The efficient transmission of correlated signals, observed at various nodes, to one or more collectors is of wide interest in various scenarios, such as sensor networks [1], and has been the subject of recent research attention. For instance, [2] discusses the spatial dependence between data according to the distribution of the nodes in the monitored area through empirical measurements. The design of efficient transmission schemes for correlated sources through orthogonal additive white Gaussian noise (AWGN) channels is a well established topic, see, e.g., [3]. In this case, the separation between source and channel coding is optimal and the ultimate performance can be achieved by means of distributed source coding (DSC) followed by independent capacity-achieving channel coding [3,4]. An alternative solution is based on the use of distributed joint source-channel coding (JSCC), where proper codes are used to encode the correlated sources. JSCC is based on the use

of standard channel codes and may represent a practical and efficient solution. In both cases, knowledge of the statistical source correlation is exploited at the joint decoder, whereas source encoding is performed separately [5]. Orthogonal multiple access schemes with an arbitrary number of correlated sources have been recently addressed in [6]. In this contribution, the asymptotic achievable region for increasing number of sources has been characterized in terms of individual channel capacities, for a specific correlation model, and pragmatic JSCC schemes have been proposed.

In this paper, we discuss various correlation models for an arbitrary number of binary sources, which may be of interest in several realistic communication scenarios. With the exception of [6], there are not many papers in the literature which discuss correlation models for a possibly large numbers of sources. In [7], the authors proposed a correlation model based on a set of linear equations in the binary field. This model is shown to be related, under special conditions, to one of the binary symmetric channels (BSC)-based models discussed in this paper. The main contribution of this paper is twofold. First, we give a unified overview on the statistical characterization of such models in terms of joint probability mass function (PMF), covariance, and joint entropy of the sources. Then,

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we derive the asymptotic entropy rate for large number of sources and show it is invariant to the considered model. This implies that the asymptotic achievable region, recently discussed in [6], which is characterized in terms of the source entropy rate, can be inferred to be independent of the specific correlation model and pragmatic joint source-channel coded schemes may be expected to have similar asymptotic behavior regardless of the model.

This paper is structured as follows. In Section 2, we present various binary source correlation models. In Section 3, we statistically characterize these models, by deriving the joint PMF of their output sequences and the corresponding covariance matrices. In Section 4, we statistically characterize these schemes in terms of their joint entropy. In Section 5, we use the joint entropy rate of these models to characterize the performance limit of orthogonal multiple access schemes transmitting correlated symbols. Finally, concluding remarks are given in Section 6.

2. Source correlation models

Consider N source nodes, possibly spatially distributed, which output (emit) binary information sequences $\mathbf{X} = (X_1, X_2, \dots, X_N)^T$, where $(\cdot)^T$ is the transpose operation. The binary information symbols are assumed to be marginally equiprobable, but correlated with each other according to a given PMF $P_{\mathbf{X}}(\mathbf{x})$, in which the N -element vector \mathbf{x} describes a possible realization of \mathbf{X} . This scenario may be representative of a sensor network in which the sensors observe N correlated physical quantities of interest. In Fig. 1, possible correlation models are shown: (a) parallel, (b) serial, and (c) mixed. In the parallel model (a), the source symbols are the output of a set of parallel BSCs, with cross-over probability $1 - \rho_\ell$, for $\ell = 1, 2, \dots, N$, denoted as BSC(ρ_ℓ), whose input is a hidden common information bit B . The ℓ -th source symbol is given by

$$X_\ell = B \oplus Z_\ell \quad (1)$$

where B is an equiprobable binary random variable, Z_ℓ are independent binary random variables with $P(Z_\ell = 0) = \rho_\ell$, $1/2 \leq \rho_\ell \leq 1$ for $\ell = 1, 2, \dots, N$, and \oplus denotes a modulo-2 sum. The random variables B and $\{Z_\ell\}_{\ell=1}^N$ are independent. Note that the random variables $\{Z_\ell\}_{\ell=1}^N$ and $\{X_\ell\}_{\ell=1}^N$ form stochastic processes in the spatial domain. Obviously, if $\rho_\ell = 0.5$ there is no correlation among the binary information symbols $\{X_\ell\}_{\ell=1}^N$, whereas if $\rho_\ell = 1$ they are identical with probability 1.

The parallel model in (1) can be seen as a special case of a parallel model with binary asymmetric channels characterized by two probabilities:

$$P(X_\ell = 0|B = 0) = \rho_{\ell,0}$$

$$P(X_\ell = 1|B = 1) = \rho_{\ell,1}.$$

In the network information theory realm, the parallel model in Fig. 1(a) is a particular instance of the Chief Executive Problem (CEO) [8]. Note, however, that in the CEO problem the detection of the hidden source B is of interest, whereas in this paper we focus on the statistical characterization of the correlated sources $\{X_\ell\}_{\ell=1}^N$. The asymmetric model is representative of wireless sensor

networking with asymmetric false alarm and missed detection probabilities, see, e.g., [9–11]. In the following, we focus on the balanced case, but a generalization of the statistical characterization presented in Sections 3 and 4 for the asymmetric parallel model is provided in Appendix A.

In Fig. 1(b), a possible serial correlation model is shown, in which the source symbols are correlated by a cascade of BSCs¹ with cross-over probability $1 - \rho_\ell$. Again the output symbols are uncorrelated for $\rho_\ell = 0.5$, whereas they are equal with probability 1 for $\rho_\ell = 1$. This model may arise in multihop relay networks, where a source symbol is delivered to a final destination through intermediate relays [12]. However, note that the paper’s focus is on the correlation structure of the data at the output of the intermediate BSCs.

A more general “mixed” case with a number m of serial branches is shown in Fig. 1(c), in which $1 - \rho_{ij}$ denotes the cross-over probability of the i th BSC on the j th branch. In this case, the correlated data at the ℓ -th, $\ell = 1, 2, \dots, N$, source on the j th branch, $j = 1, 2, \dots, M$, can be expressed as

$$X_{\ell j} = B \oplus \underbrace{\sum_{i=1}^{\ell} Z_{ij}}_{\triangleq Z'_{\ell j}} = B \oplus Z'_{\ell j}$$

where the symbol $\sum_{i=1}^{\ell}$ denotes modulo-2 sums. The random variable $Z'_{\ell j}$ can be easily characterized by its distribution [13, Lemma 4.1]

$$p_{\ell j} \triangleq P(Z'_{\ell j} = 0) = \frac{1}{2} \left[1 + \prod_{i=1}^{\ell} (2\rho_{ij} - 1) \right]. \quad (2)$$

Note that for $M = 1$, this mixed model reduces to the serial one of Fig. 1(b) and the index j in (2) can be dropped.

A model based on a set of binary linear equations was considered in [7]:

$$\mathbf{A}\mathbf{X} = \mathbf{Z} \quad (3)$$

where \mathbf{A} is a binary matrix (whose entries are equal to either 0 or 1) defining the set of equations and $\mathbf{Z} = (Z_1, Z_2, \dots, Z_N)^T$ is a vector of independent Bernoulli binary random variables with parameters $\rho_\ell = P(Z_\ell = 0)$. Note that matrix operations are performed in the binary field.

If \mathbf{A} is such that $A_{\ell\ell} = 1$, $A_{\ell,\ell-1} = 1$, $A_{\ell k} = 0$ for $k > \ell$, and $\rho_1 = 1/2$, (3) is equivalent to the serial model in Fig. 1(b).² In fact, as a special case, (3) may encompass a set of recursive equations of the form

$$\sum_{i=0}^{\min\{D,\ell-1\}} A_{\ell,\ell-i} X_{\ell-i} = Z_\ell \quad (4)$$

¹ The first BSC does not play any role in the serial model and could be omitted—it is kept for notational consistency with the other models.

² Note that the parallel model (1) cannot be directly described by (3) since no structure of \mathbf{A} can be found. However, the parallel model could be described by a modified system $\mathbf{A}\mathbf{X} \oplus \mathbf{B} = \mathbf{Z}$, where \mathbf{B} is a size- N vector with all elements equal to B and $\mathbf{A} = \mathbf{I}_N$, being \mathbf{I}_N the identity matrix of size N . Similar considerations hold for the mixed model, which is based on a combination of parallel and serial models.

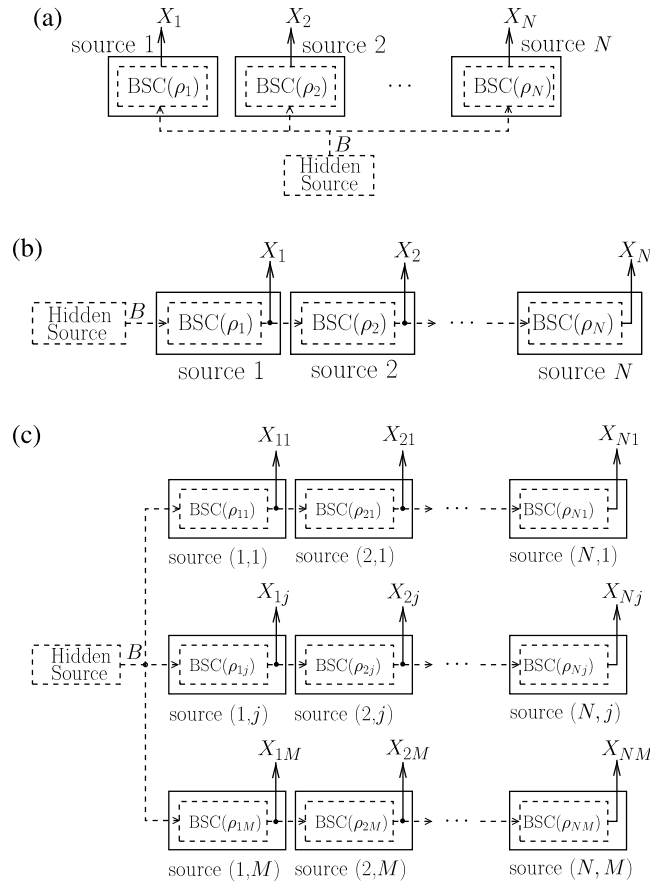


Fig. 1. Considered correlation models: (a) parallel, (b) serial, and (c) mixed.

in which D is the recursion depth, $A_{\ell\ell} = 1$, $A_{\ell k} = 0$ or 1 for $k = \ell - \min\{D, \ell - 1\}, \dots, \ell - 1$, and $A_{\ell k} = 0$ for $k > \ell$. Since we are considering binary random variables, (4) can be also rewritten as

$$X_\ell = \begin{cases} Z_\ell & \ell = 1 \\ Z_\ell \oplus \sum_{i=1}^{\min\{D, \ell-1\}} A_{\ell, \ell-i} X_{\ell-i} & \ell = 2, 3, \dots, N \end{cases} \quad (5)$$

which for $D = 1$ and $\rho_1 = 1/2$ reduces to the model in Fig. 1(b). The model in (5) describes a recursive binary filter with input Z_ℓ and output X_ℓ . Note that the recursive model is characterized by a matrix \mathbf{A} with Toeplitz structure.

If the matrix \mathbf{A} is invertible in the binary field, then

$$\mathbf{X} = \mathbf{A}^{-1} \mathbf{Z} \quad (6)$$

and, therefore, there exists a one-to-one correspondence between \mathbf{X} and \mathbf{Z} . A case where the inverse exists is the recursive model in (4), as it is shown in Appendix B, where a few cases of interest are also analyzed. For simplicity, in the rest of the paper we will assume that the matrix \mathbf{A} is invertible. Note that, invertibility may lead to uncorrelated binary sources in the special case of matrices with constant row weight equal to d_c , as shown in [14].

3. Statistical characterization

According to the parallel correlation model (1) in Fig. 1(a), the joint PMF of the information symbols at the output of the N nodes can be computed. By straightforward manipulations, one can show that

$$P_{\mathbf{X}}(\mathbf{x}) = \sum_{b=0,1} P_{\mathbf{X}}(\mathbf{x}|B=b) P_B(b) = \frac{1}{2} \left[\prod_{\ell \in \mathcal{S}_0} \rho_\ell \prod_{k \in \mathcal{S}_1} (1 - \rho_k) + \prod_{\ell \in \mathcal{S}_0} (1 - \rho_\ell) \prod_{k \in \mathcal{S}_1} \rho_k \right] \quad (7)$$

where \mathcal{S}_0 and \mathcal{S}_1 is a partition of the set $\{1, 2, \dots, N\}$ specifying the positions of zeros and ones in \mathbf{x} , respectively. In the special case of $\rho_\ell = \rho$, $\ell = 1, 2, \dots, N$, one obtains

$$P_{\mathbf{X}}(\mathbf{x}) = \frac{1}{2} \left[\rho^{n_z} (1 - \rho)^{N - n_z} + (1 - \rho)^{n_z} \rho^{N - n_z} \right] \quad (8)$$

where n_z is the number of zeros in \mathbf{x} .

In the serial case of Fig. 1(b), using the chain rule, one has

$$P_{\mathbf{X}}(\mathbf{x}) = \sum_{b=0,1} P_{\mathbf{X}}(\mathbf{x}|B=b) P_B(b) = \sum_{b=0,1} P_B(b) P_{X_1}(x_1|B=b) \prod_{\ell=2}^N P_{X_\ell}(x_\ell|X_{\ell-1}=x_{\ell-1}) \quad (9)$$

where we have used the fact that, given $X_{\ell-1}$, X_ℓ is conditionally independent from the previous source symbols $X_{\ell-2}, \dots, X_1$ and B . After further simple manipulations, one can write

$$P_{\mathbf{X}}(\mathbf{x}) = \frac{1}{2} \prod_{\ell \in \mathcal{L}_0} \rho_\ell \prod_{k \in \mathcal{L}_1} (1 - \rho_k)$$

where \mathcal{L}_0 and \mathcal{L}_1 is a partition of the set $\{2, 3, \dots, N\}$ specifying the positions where $x_\ell = x_{\ell-1}$ and $x_\ell \neq x_{\ell-1}$, respectively. In the special case of $\rho_\ell = \rho$, $\ell = 2, 3, \dots, N$, denoting as n'_z the cardinality of the set \mathcal{L}_0 , one obtains

$$P_{\mathbf{X}}(\mathbf{x}) = \frac{1}{2} \rho^{n'_z} (1 - \rho)^{N-n'_z-1}.$$

In the general mixed case of Fig. 1(c), let us denote as

$$X_{1j}^N \triangleq (X_{1j}, \dots, X_{Nj}) \quad j = 1, 2, \dots, M$$

the set of the N source symbols on the j th branch. One can generalize the result of the serial case by writing:

$$\begin{aligned} P_{\mathbf{X}}(\mathbf{x}) &= \sum_{b=0,1} P_{\mathbf{X}}(\mathbf{x}|B=b)P_B(b) \\ &= \sum_{b=0,1} \prod_{j=1}^M P_{X_{1j}^N}(x_{1j}^N|B=b)P_B(b) \\ &= \sum_{b=0,1} \prod_{j=1}^M P_B(b)P_{X_{1j}}(x_{1j}|B=b) \\ &\quad \times \prod_{\ell=2}^N P_{X_{\ell j}}(x_{\ell j}|X_{\ell-1,j} = x_{\ell-1,j}) \end{aligned}$$

where, in the second line, we have used the fact that, conditionally on B , the branches are independent, and the last line is equivalent to (9) applied to each branch.

Finally, the PMF for correlated sources following the model in [7] can be characterized, by resorting to (3), as

$$P_{\mathbf{X}}(\mathbf{x}) = P_{\mathbf{Z}}(\mathbf{A}\mathbf{x}). \quad (10)$$

Using simple manipulations, we have

$$P_{\mathbf{X}}(\mathbf{x}) = \prod_{\ell \in \mathcal{S}'_0} \rho_\ell \prod_{k \in \mathcal{S}'_1} (1 - \rho_k)$$

where \mathcal{S}'_0 and \mathcal{S}'_1 is a partition of the set $\{1, 2, \dots, N\}$ specifying the positions of zeros and ones in $\mathbf{A}\mathbf{x}$, respectively.

The covariance matrix for the considered correlation models is defined as

$$\mathbf{C}_X = \mathbf{R}_X - \boldsymbol{\mu}_X \boldsymbol{\mu}_X^T.$$

For the parallel and serial models, the elements of the correlation matrix \mathbf{R}_X and the mean vector $\boldsymbol{\mu}_X$ are, respectively,

$$\begin{aligned} R_{ik} &= \mathbb{E}[X_i X_k] \\ \mu_i &= \mathbb{E}[X_i] \quad i, k = 1, 2, \dots, N. \end{aligned}$$

Note that $R_{ik} = \mathbb{E}[X_i X_k] = P(X_i = X_k = 1)$, since the data are binary. The elements of the vector $\boldsymbol{\mu}_X$ can be obtained as

$$\mu_i = \frac{1}{2}.$$

\mathbf{R}_X depends on the considered correlation model, but in all cases

$$R_{ii} = \mathbb{E}[X_i^2] = 1 \cdot P(X_i = 1) = \frac{1}{2}$$

since, for all considered models, $P(X_i = 1) = 0.5$.

In the parallel case of Fig. 1(a), for $i, k = 1, 2, \dots, N$ ($i \neq k$) one obtains

$$\begin{aligned} \mathbb{E}[X_i X_k] &= \frac{1}{2} \sum_{b=0,1} P(X_i = X_k = 1|B=b) \\ &= \frac{1}{2} [\rho_i \rho_k + (1 - \rho_i)(1 - \rho_k)] \\ &= \frac{1}{2} [1 - (\rho_i + \rho_k) + 2\rho_i \rho_k] \end{aligned} \quad (11)$$

which reduces, for $\rho_\ell = \rho$, $\ell = 1, 2, \dots, N$, to

$$\mathbb{E}[X_i X_k] = \frac{1}{2} [1 - 2\rho + 2\rho^2].$$

Note that if $\rho = 1/2$, $C_{ik} = 1/4$ for $i = k$ and zero otherwise, i.e., data are uncorrelated.

For the serial correlation model in Fig. 1(b) and $i \neq k$, one can write

$$\begin{aligned} P(X_i = X_k = 1) &= P(X_k = 1|X_i = 1)P(X_i = 1) \\ &= \frac{1}{2} P(X_k = 1|X_i = 1) \end{aligned}$$

where the conditional probability $P(X_k = 1|X_i = 1)$ can be computed noting that $X_i = X_k = 1$ if the BSCs of indices $i + 1, i + 2, \dots, k$ flip an even number of times. Therefore, by arguments similar to those in [13, Lemma 4.1] we obtain

$$P(X_k = 1|X_i = 1) = \frac{1}{2} \left[1 + \prod_{\ell=i+1}^k (2\rho_\ell - 1) \right]$$

and, therefore,

$$\mathbb{E}[X_i X_k] = \frac{1}{4} \left[1 + \prod_{\ell=i+1}^k (2\rho_\ell - 1) \right]. \quad (12)$$

For the special case of $\rho_\ell = \rho$, $\ell = 2, 3, \dots, N$, denoting the number of hops in the branch of BSCs as $L = |i - k|$, (12) reduces to

$$\mathbb{E}[X_i X_k] = \frac{1}{4} [1 + (2\rho - 1)^L].$$

Note that if $\rho = 1/2$, $C_{ik} = 1/4$ for $i = k$ and zero otherwise, i.e., data are uncorrelated.

In the mixed scenario of Fig. 1(c), the covariance matrix has size $NM \times NM$. The element C_{ik} , for $i, k = 1, 2, \dots, NM$, can be defined as the covariance between the source symbols $X_{\ell_1 m_1}$ and $X_{\ell_2 m_2}$, where

$$i = (m_1 - 1)N + \ell_1 \quad k = (m_2 - 1)N + \ell_2.$$

In particular, the following two cases may occur:

- if $m_1 = m_2$, i.e., $X_{\ell_1 m_1}$ and $X_{\ell_2 m_1}$ belong to the same branch, the result in (12) can be applied;
- if $m_1 \neq m_2$, i.e., $X_{\ell_1 m_1}$ and $X_{\ell_2 m_2}$ belong to different branches, the results for two sources in a parallel scheme can be applied replacing ρ_i and ρ_k in (11) with $p_{\ell_1 m_1}$ and $p_{\ell_2 m_2}$, respectively, as defined in (2).

Obviously, it is still verified that, if $\rho_{ik} = \rho = 1/2$, $C_{ik} = 1/4$ for $i = k$ and zero otherwise, i.e., data are uncorrelated.

Finally, the covariance matrix can be also considered for the correlation model (3) based on a set of binary linear equations. Using (6), one can write

$$\begin{aligned} \mathbf{C}_X &= \mathbf{R}_X - \boldsymbol{\mu}_X \boldsymbol{\mu}_X^T \\ &= \mathbb{E}[\mathbf{A}^{-1} \mathbf{Z} \mathbf{Z}^T (\mathbf{A}^{-1})^T] - \mathbb{E}[\mathbf{A}^{-1} \mathbf{Z}] \mathbb{E}[\mathbf{Z}^T (\mathbf{A}^{-1})^T]. \end{aligned}$$

Note that the elements of vectors and matrices in the second line are random variables which can be either 0 or 1, i.e., they are Bernoulli random variables. Since it is known that the mean value of a Bernoulli random variable is equal to the probability that the random variable itself is equal to 1, one has

$$\begin{aligned} \mathbf{C}_X &= \mathcal{P}(\mathbf{A}^{-1} \mathbf{Z} \mathbf{Z}^T (\mathbf{A}^{-1})^T = \mathbf{J}_N) \\ &\quad - \mathcal{P}(\mathbf{A}^{-1} \mathbf{Z} = \mathbf{1}_N) \mathcal{P}(\mathbf{Z}^T (\mathbf{A}^{-1})^T = \mathbf{1}_N) \end{aligned}$$

where \mathbf{J}_N is the all-one matrix of size $N \times N$, $\mathbf{1}_N$ is the all-one column vector of length N , and $\mathcal{P}(\cdot)$ denotes element-wise probability. Note that a closed-form solution for \mathbf{C}_X is not readily available as it depends on the particular structure of \mathbf{A} .

An overview of the statistical characterization of the considered models is summarized in Table 1.

Fig. 2 shows the coefficients of the first row of the covariance matrix \mathbf{C}_X for $N = 5$ sources, $\rho_\ell = \rho_{ik} = \rho$, and the three models: (a) parallel, (b) serial, and (c) mixed with $M = 2$. Two values of ρ are considered: 0.7 (left bars in each figure) and 0.95 (right bars in each figure). Only the first row is considered, since for parallel and serial models with constant ρ the covariance matrix is symmetric and Toeplitz. In the mixed case, instead, for constant ρ , \mathbf{C}_X has the following block Toeplitz structure

$$\mathbf{C}_X = \begin{bmatrix} \mathbf{C}_1 & \mathbf{C}_2 & \cdots & \mathbf{C}_M \\ \mathbf{C}_2^T & \mathbf{C}_1 & \cdots & \mathbf{C}_{M-1} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{C}_M^T & \mathbf{C}_{M-1}^T & \cdots & \mathbf{C}_1 \end{bmatrix}$$

where \mathbf{C}_i is a Toeplitz matrix, of size $N \times N$, representing the covariance between sources on branches with separation $i - 1$. In other words, zero separation means that the sources are in the same branch, separation 1 means that sources are on adjacent branches, and so on. In this case as well, the first row is sufficient to characterize the entire matrix. In the figure, one can observe that the first coefficient is equal to the symbol variance ($1/4$) in all cases. Moreover, in the parallel model (a), all coefficients C_{1k} for k from 2 to 5 are equal due to the fact that the pairwise probabilities are identical, regardless of the source index. Note also that the higher the value of ρ , the higher the covariance elements, since data are more and more correlated. In the serial model (b), the covariance decreases with k , since a larger number of BSCs is present between the sources, which become more and more uncorrelated. In the mixed model (c), recall that indices from $k = 1$ to $k = 5$ correspond to the first branch of BSCs, whereas indices from $k = 6$ to $k = 10$ correspond to the second branch. As expected, symbols in the second branch are less correlated with the first symbol X_{11} , than those in the first one at similar depth.

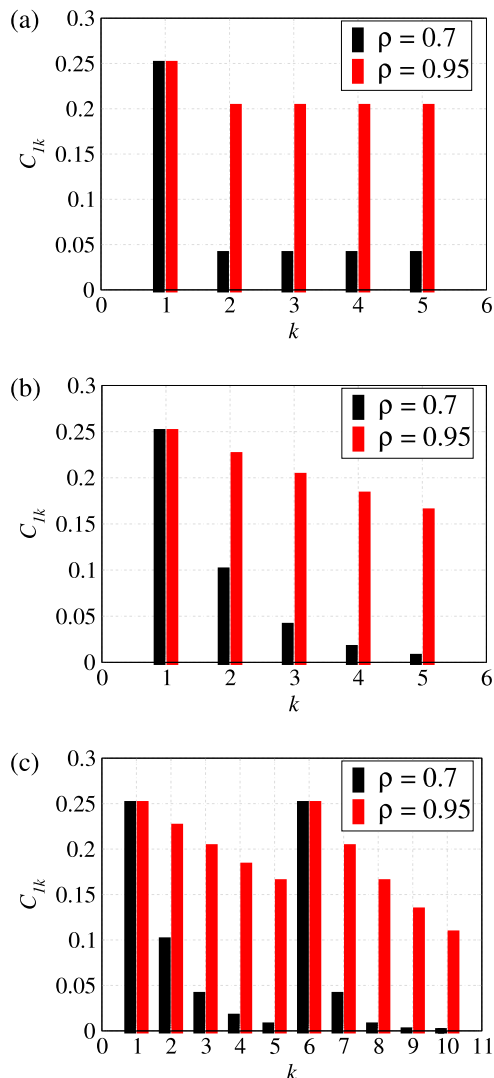


Fig. 2. Coefficients of the first row of the covariance matrix \mathbf{C}_X for $N = 5$ sources and three models: (a) parallel, (b) serial, and (c) mixed with $M = 2$. Two values of ρ are considered: 0.7 (left bars in each figure) and 0.95 (right bars in each figure).

An alternative view of the covariance matrix is the histogram of its values shown in Fig. 3 for $N = 5$ sources and two models: (a) parallel and (b) serial. Two values of $\rho_\ell = \rho$ are considered: 0.7 and 0.95. One can observe that in the parallel model, only two values of C_{ik} are allowed, since the pairwise probabilities are the same for any pair of sources. Moreover, for high correlation (e.g., $\rho = 0.95$) larger values than those for small correlation (e.g., $\rho = 0.7$) are obtained, which is in agreement with the fact that $C_{ik} \rightarrow 0$ for $i \neq k$ if $\rho \rightarrow 1/2$.

4. Source entropy rate

To compute the joint entropy of the N sources for the considered correlation models, we denote it as $H(X_1^N)$, where the notation X_a^b indicates the sequence

Table 1
Summary of the statistical characterization for all considered models.

Model	PMF	Covariance
Parallel	$P_{\mathbf{X}}(\mathbf{x}) = \frac{1}{2} \left[\prod_{\ell \in \delta_0} \rho_\ell \prod_{k \in \delta_1} (1 - \rho_k) + \prod_{\ell \in \delta_0} (1 - \rho_\ell) \prod_{k \in \delta_1} \rho_k \right]$	$C_{ik} = \begin{cases} \frac{1}{4} & i = k \\ \frac{1}{2} \left[\frac{1}{2} - (\rho_i + \rho_k) + 2\rho_i\rho_k \right] & i \neq k \end{cases}$
Serial	$P_{\mathbf{X}}(\mathbf{x}) = \frac{1}{2} \prod_{\ell \in \mathcal{L}_0} \rho_\ell \prod_{k \in \mathcal{L}_1} (1 - \rho_k)$	$C_{ik} = \begin{cases} \frac{1}{4} & i = k \\ \frac{1}{4} \prod_{\ell=i+1}^k (2\rho_\ell - 1) & i \neq k \end{cases}$
Mixed $\mathbf{AX} = \mathbf{Z}$	$P_{\mathbf{X}}(\mathbf{x}) = \sum_{b=0,1} \prod_{j=1}^M P_B(b) P_{X_{1j}}(x_{1j} B=b) \prod_{\ell=2}^N P_{X_{\ell j}}(x_{\ell j} X_{\ell-1,j} = x_{\ell-1,j})$ $P_{\mathbf{X}}(\mathbf{x}) = \prod_{\ell \in \delta_0'} \rho_\ell \prod_{k \in \delta_1'} (1 - \rho_k)$	Combination of parallel and serial depending on i, k $C_{\mathbf{X}} = \mathcal{P}(\mathbf{A}^{-1}\mathbf{Z}\mathbf{Z}^T(\mathbf{A}^{-1})^T = \mathbf{J}_N) - \mathcal{P}(\mathbf{A}^{-1}\mathbf{Z} = \mathbf{1}_N) \mathcal{P}(\mathbf{Z}^T(\mathbf{A}^{-1})^T = \mathbf{1}_N)$

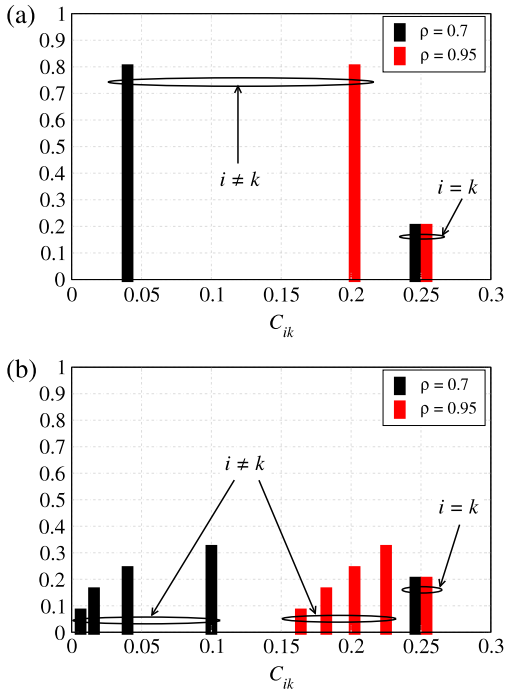


Fig. 3. Histogram of the values of the matrix $C_{\mathbf{X}}$ for $N = 5$ sources and two models: (a) parallel and (b) serial. Two values of ρ are considered: 0.7 and 0.95.

$(X_a, X_{a+1}, \dots, X_b)$, $a \leq b$. Let us consider the joint entropy of X_1^N and B :

$$H(B, X_1^N) = H(B) + H(X_1^N|B). \quad (13)$$

For the parallel model (1) of Fig. 1(a), one has

$$H(X_1^N|B) = \sum_{\ell=1}^N H_b(\rho_\ell)$$

where $H_b(\rho_\ell)$ is the entropy of a binary random variable with parameter ρ_ℓ . Since B is a uniformly distributed binary random variable, $H(B) = H_b(0.5) = 1$. Therefore, one obtains:

$$H(B, X_1^N) = 1 + \sum_{\ell=1}^N H_b(\rho_\ell).$$

As in (13), it is also possible to write

$$H(B, X_1^N) = H(X_1^N) + H(B|X_1^N) = 1 + \sum_{\ell=1}^N H_b(\rho_\ell)$$

and, therefore,

$$H(X_1^N) = 1 + \sum_{\ell=1}^N H_b(\rho_\ell) - H(B|X_1^N). \quad (14)$$

By definition of entropy, the last term is non negative and the following upper bound (UB) is obtained

$$H(X_1^N) \leq 1 + \sum_{\ell=1}^N H_b(\rho_\ell). \quad (15)$$

Moreover, since conditioning reduces entropy [15], it also follows that

$$H(B|X_1^N) \leq H(B|X_1) = H_b(\rho_1).$$

Using this in (14), one obtains the lower bound (LB):

$$\begin{aligned} H(X_1^N) &\geq 1 + \sum_{\ell=1}^N H_b(\rho_\ell) - H_b(\rho_1) \\ &= 1 + \sum_{\ell=2}^N H_b(\rho_\ell). \end{aligned} \quad (16)$$

Combining (15) and (16) and analyzing the limit for large number of sources, one obtains

$$\lim_{N \rightarrow +\infty} \frac{H(X_1^N)}{N} = \bar{H}_b \quad (17)$$

where

$$\bar{H}_b \triangleq \lim_{N \rightarrow +\infty} \frac{1}{N} \sum_{\ell=1}^N H_b(\rho_\ell)$$

in which the limit exists since $0 \leq H_b(\rho_\ell) \leq 1$. This quantity can be interpreted as the limit average entropy of the cascade of N BSCs and (17) can be interpreted as the asymptotic entropy rate of the correlated sources, namely the limit average source entropy. Eq. (17) reduces, for the special case $\rho_\ell = \rho$, $\ell = 1, 2, \dots, N$, to

$$\bar{H}_b = \lim_{N \rightarrow +\infty} \frac{1}{N} \sum_{\ell=1}^N H_b(\rho_\ell) = H_b(\rho).$$

This result can be also obtained by observing that X_1^N is a stationary binary random process, whose entropy rate is well-known [15, Ch. 4]. This limit for the special case of constant ρ has been also derived in [6].

Consider now the joint entropy of the serial correlated source model in Fig. 1(b). Using the chain rule for entropy, one can easily compute the joint entropy as

$$\begin{aligned} H(X_1^N) &= H(X_1) + \sum_{\ell=2}^N H(X_\ell | X_1^{\ell-1}) \\ &= H(X_1) + \sum_{\ell=2}^N H(X_\ell | X_{\ell-1}) \\ &= 1 + \sum_{\ell=2}^N H_b(\rho_\ell). \end{aligned} \quad (18)$$

Analyzing the limit for large number of sources, one obtains

$$\lim_{N \rightarrow +\infty} \frac{H(X_1^N)}{N} = \bar{H}_b \quad (19)$$

and in the special case of $\rho_\ell = \rho$, $\ell = 2, 3, \dots, N$:

$$\bar{H}_b = \lim_{N \rightarrow +\infty} \frac{1}{N} \left[1 + \sum_{\ell=2}^N H_b(\rho_\ell) \right] = H_b(\rho).$$

Note that the parallel and serial models have equal asymptotic source entropy rate for any given sequence $\{\rho_\ell\}_{\ell=1}^N$.

In the general mixed case of Fig. 1(c), it can be shown (see Appendix C for the proof) that

$$\lim_{N \rightarrow +\infty} \frac{H(X_{11}^N, \dots, X_{1M}^N)}{MN} = \bar{\bar{H}}_b \quad (20)$$

where

$$\bar{\bar{H}}_b \triangleq \lim_{N \rightarrow +\infty} \frac{1}{MN} \sum_{j=1}^M \sum_{\ell=1}^N H_b(\rho_{\ell j}).$$

We remark that the limit exists since each term in the summation is limited to the interval $[0, 1]$. Eq. (20) reduces, for the special case of $\rho_{\ell j} = \rho$, $\ell = 1, 2, \dots, N$ and $j = 1, 2, \dots, M$, to

$$\bar{\bar{H}}_b = \lim_{N \rightarrow +\infty} \frac{1}{MN} \sum_{j=1}^M \sum_{\ell=1}^N H_b(\rho) = H_b(\rho).$$

Note that similar considerations can be also carried out if N is kept fixed and M grows to infinity or if both N and M become arbitrarily large.

We finally analyze the entropy rate of the source correlation model given by a set of linear equations in (3). Using (10) and the assumption of invertibility of \mathbf{A} , it can be easily shown that [15]

$$H(X_1^N) = H(Z_1^N) = \sum_{\ell=1}^N H_b(\rho_\ell).$$

Therefore:

$$\lim_{N \rightarrow +\infty} \frac{H(X_1^N)}{N} = \bar{H}_b$$

which reduces, for the special case of $\rho_\ell = \rho$, $\ell = 1, 2, \dots, N$, to

$$\bar{H}_b = \lim_{N \rightarrow +\infty} \frac{1}{N} \sum_{\ell=1}^N H_b(\rho_\ell) = H_b(\rho).$$

This shows that the source entropy rate is asymptotically the same for the linear equation-based correlation model as well, hence for all the considered correlation models.

The convergence of the entropy rates of the considered models can be analyzed in terms of the difference between the average source entropy for finite N and the asymptotic value. In particular, in the parallel and serial models we define

$$\epsilon \triangleq \frac{H(X_1^N)}{N} - \bar{H}_b.$$

In the mixed model, instead, this difference is also function of M and can be defined as

$$\epsilon \triangleq \frac{H(X_{11}^N, \dots, X_{1M}^N)}{MN} - \bar{\bar{H}}_b.$$

In Fig. 4, bounds on ϵ are shown, as functions of N , for parallel or serial models and two values of ρ (assumed equal for all BSCs): 0.7 and 0.95. Note that lower and upper bounds on ϵ can be obtained using the LB and UB on the joint entropy for the parallel model, respectively. In particular the LB for the parallel case and the exact value of ϵ for the serial model are identical, due to the fact that the right-hand side of (16) and (18) coincide for any N in the special case of constant ρ . Moreover, the UB on ϵ for the parallel model is the same, regardless of the value of ρ , and the curves overlap for both values of ρ . In both cases, ϵ converges to zero as $1/N$.³ Furthermore, the tightness of the bounds increases with the number of correlated sources, since UB and LB become closer to each other. However, the convergence of the LB degrades with increasing values of ρ . Moreover, the mixed model has the same trend for ϵ , since it decreases with $1/(MN)$ for any fixed value of M .

In Fig. 5, bounds on ϵ are shown, as functions of M , for the mixed model, two values of ρ (namely, 0.7 and 0.95), and two values of N (namely, 10 and 50). Note that similar considerations as those relative to Fig. 4 hold in this case for fixed values of N and letting M go to infinity.

5. Transmission of correlated symbols in orthogonal multiple access schemes

The statistical characterization of the correlation models can be useful in the design and performance analysis of orthogonal multiple access schemes with correlated sources. In these schemes, each node independently encodes, by a joint source-channel code with rate r , the source symbols and transmits them through an orthogonal multiple access channel. At the receiver side, data are

³ This convergence rate can be also obtained for the model given by a set of linear equations when \mathbf{A} is such that this model is equivalent to the serial one.

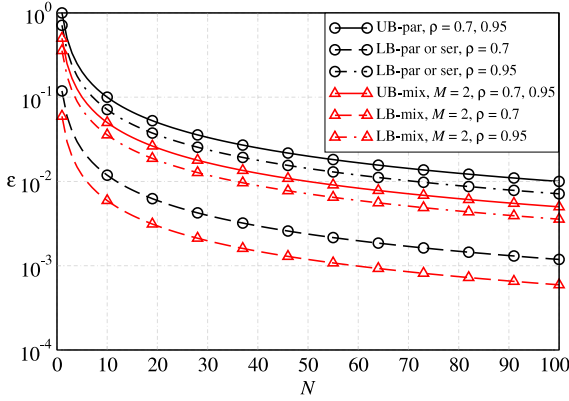


Fig. 4. Bounds on ϵ , as functions of N , for parallel (par), serial (ser), and mixed (mix) ($M = 2$) models and two values of ρ : 0.7 and 0.95.

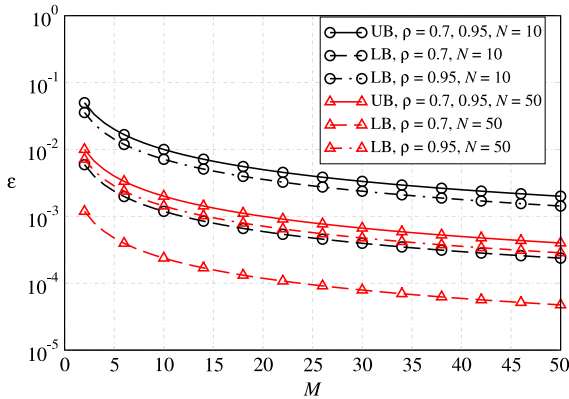


Fig. 5. Bounds on ϵ , as functions of M , for the mixed model, two values of ρ (namely, 0.7 and 0.95), and two values of N (namely, 10 and 50).

decoded by properly taking into account the source correlation to improve the overall system performance.

In this scenario, the source PMF is helpful to design proper iterative receivers, as shown in [6]. The computation of the source entropy rate discussed in Section 4 plays, instead, an important role in determining the asymptotic achievable region of orthogonal multiple access schemes with correlated sources. In the following, we will shortly discuss the impact of the correlation model on the achievable region of orthogonal multiple access schemes.

Recent work in [6] has proposed a characterization of the achievable region of orthogonal multiple access schemes with correlated sources, based on the computation of joint and conditional entropies of the sources, for an arbitrary value of N . According to [6], the achievable region, in the space of individual channel capacity values $\{\lambda_\ell\}_{\ell=1}^N$, is specified by the intersection of the following inequalities:

$$\sum_{\ell \in \mathcal{S}} \lambda_\ell \geq r H(X(\mathcal{S})|X(\mathcal{S}_c)) \quad (21)$$

for all $\mathcal{S} \subseteq \{1, 2, \dots, N\}$, in which $X(\mathcal{S}) = \{X_i : i \in \mathcal{S}\}$ and \mathcal{S}_c denotes the complementary set of \mathcal{S} . Note that $H(X(\mathcal{S})|X(\mathcal{S}_c))$ is the conditional entropy of the sources with index in \mathcal{S} given the remaining ones. Two

characteristic operational points, denoted as “balanced” and “unbalanced,” are of interest.

The *balanced* characteristic point refers to the case where all source symbols are transmitted at a rate equal to the same single-channel capacity, i.e., $\lambda_{\text{bal}} = \lambda_1 = \lambda_2 = \dots = \lambda_N$. This common value, is equal to

$$\lambda_{\text{bal}} \triangleq r \frac{H(X_1^N)}{N}. \quad (22)$$

The *unbalanced* case, instead, refers to the portion of the achievable region characterized as follows: $N - 1$ sources, e.g., sources from 1 to $N - 1$, are associated with sufficiently large values of λ_i , $i = 1, 2, \dots, N - 1$. In this case, λ_{unb} is the smallest value of λ_N such that the operational point lies on the border of the achievable region and it is equal to

$$\lambda_{\text{unb}} \triangleq r H(X_N|X_1^{N-1}). \quad (23)$$

In [6], it is shown that for the parallel model in Fig. 1(a) and $\rho_\ell = \rho$, $\ell = 1, 2, \dots, N$, the following facts hold

$$\lambda_{\text{bal}} \geq \lambda_{\text{unb}} \quad \forall N$$

$$\lim_{N \rightarrow +\infty} \lambda_{\text{unb}} = \lim_{N \rightarrow +\infty} \lambda_{\text{bal}} \triangleq \lambda_{\text{lim}} = r H_b(\rho). \quad (24)$$

This characterization of the achievable orthogonal multiple access region is simple but effective. In particular, (24) tells us that, when N increases, the achievable region tends to a hyperoctant and the system operational points become equal. Therefore, one can devise joint source-channel coded schemes for any of these operational points, since they guarantee the same achievable rate of other operational points for a sufficiently large number of sources. Hence, code design can be based on the most convenient operational point, e.g., the one which guarantees less complexity.

A natural question, not discussed in [6], is the following: are these results dependent on the considered correlation model? In general, the entropy of the binary sources may depend on the model considered for correlation. However, we have shown in Section 4 that the asymptotic entropy rate is the same for all the considered models. We can, therefore, conclude that the achievable region of orthogonal multiple access schemes is asymptotically invariant to the considered correlation model as the number of sources increases. Moreover, from Figs. 4 and 5 it can be observed that the convergence trend is the same for all considered models. Hence, it may be expected that joint source-channel coding schemes devised for one of these models (see, e.g., the turbo codes considered in [6] for the parallel model) have similar asymptotic performance for the other models as the number of sources increases.

6. Conclusions

In this paper, we have discussed and analyzed a few binary source correlation models. In particular, we have statistically characterized these models, in terms of joint PMF, covariance, and joint entropy of the source correlated sequence. Moreover, we have computed the asymptotic entropy rate for large number of sources, showing that a common value is obtained for all considered models. This result generalizes [6], because the asymptotic

achievable region of orthogonal multiple access schemes for large numbers of sources is shown to be invariant to the considered correlation model. Therefore, one can conjecture that joint source-channel coding schemes, which exploit the correlation at the receiver, may be expected to have similar asymptotic behavior regardless of the model.

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Appendix A. Statistical characterization of a parallel model with asymmetric probabilities

In this Appendix, we present a generalization of the statistical characterization for a parallel model in which the correlated symbols are generated using binary asymmetric channels. In this scenario, the information data at the ℓ -th source, $\ell = 1, 2, \dots, N$ is still given by $X_\ell = B \oplus Z_\ell$ with

$$P(X_\ell = 0|B = 0) = \rho_{\ell,0}$$

$$P(X_\ell = 1|B = 1) = \rho_{\ell,1}.$$

Note that for $\rho_{\ell,0} = \rho_{\ell,1}$, $\ell = 1, 2, \dots, N$, the derivation for the symmetric scenario can be obtained. In this asymmetric case, the source symbols are in general not equiprobable. In fact:

$$\begin{aligned} P(X_\ell = 0) &= \sum_{b=0,1} P(X_\ell = 0|B = b)P(B = b) \\ &= \frac{1}{2}\rho_{\ell,0} + \frac{1}{2}(1 - \rho_{\ell,1}) \\ &= \frac{1}{2}[1 + \rho_{\ell,0} - \rho_{\ell,1}] \end{aligned}$$

and, similarly,

$$P(X_\ell = 1) = 1 - P(X_\ell = 0) = \frac{1}{2}[1 + \rho_{\ell,1} - \rho_{\ell,0}].$$

As in (7), the source PMF can be expressed as

$$\begin{aligned} P_{\mathbf{X}}(\mathbf{x}) &= \sum_{b=0,1} P_{\mathbf{X}}(\mathbf{x}|B = b)P_B(b) \\ &= \frac{1}{2} \left[\prod_{\ell \in \mathcal{S}_0} \rho_{\ell,0} \prod_{k \in \mathcal{S}_1} (1 - \rho_{k,0}) + \prod_{\ell \in \mathcal{S}_0} (1 - \rho_{\ell,1}) \prod_{k \in \mathcal{S}_1} \rho_{k,1} \right]. \end{aligned}$$

In order to derive the covariance matrix, one has to determine the following quantities:

$$\mu_i = \mathbb{E}[X_i]$$

$$R_{ik} = \mathbb{E}[X_i X_k].$$

Since $\{X_i\}$ are i.i.d. Bernoulli random variables,

$$\mu_i = 1 \cdot P(X_i = 1) = \frac{1}{2}[1 + \rho_{i,1} - \rho_{i,0}].$$

The elements of the correlation matrix, instead, can be obtained as follows. For $i = k$, one has

$$R_{ii} = \mathbb{E}[X_i^2] = 1 \cdot P(X_i = 1) = \mu_i$$

whereas, for $i \neq k$,

$$\begin{aligned} R_{ik} &= 1 \cdot P(X_i = X_k = 1) \\ &= \frac{1}{2} \sum_{b=0,1} P(X_i = X_k = 1|B = b) \\ &= \frac{1}{2} [\rho_{i,1}\rho_{k,1} + (1 - \rho_{i,1})(1 - \rho_{k,1})]. \end{aligned}$$

Finally, the joint entropy of the N sources can be derived using the approach in Section 4. In particular, it can be observed that

$$H(X_\ell|B) = \frac{1}{2} [H_b(\rho_{\ell,0}) + H_b(\rho_{\ell,1})].$$

Therefore, the entropy rate for large number of sources is

$$\lim_{N \rightarrow +\infty} \frac{H(X_1^N)}{N} = \tilde{H}_b$$

where

$$\tilde{H}_b \triangleq \lim_{N \rightarrow +\infty} \frac{1}{2N} \sum_{\ell=1}^N [H_b(\rho_{\ell,0}) + H_b(\rho_{\ell,1})].$$

Since the joint PMF, the matrix covariance, and source entropy rate have the same structure of the symmetric model in Fig. 1(a), the conclusions drawn for the symmetric case can be extended to the asymmetric one. All the results, in terms of joint PMF, covariance, and source entropy rate, for the parallel model in Fig. 1(a) can be obtained by setting $\rho_{\ell,0} = \rho_{\ell,1} = \rho_\ell$, $\ell = 1, 2, \dots, N$ in the above equations.

Appendix B. Existence of the inverse matrix in the linear correlation model (3)

The derivations for the model (3) are based on the assumption of invertibility of matrix \mathbf{A} . We now present a few special cases of matrix \mathbf{A} in (3) where the inverse matrix exists. In particular, we consider two main classes: (i) matrices associated with the recursive model in (4) and (ii) circulant matrices. To show the invertibility of these matrices, we can prove that their determinants are non-zero in the binary field.

The coefficients of matrix \mathbf{A} for the recursive model in (4) are the following:

$$A_{\ell k} = \begin{cases} 1 & \text{for } k = \ell \\ 0 \text{ or } 1 & \text{for } k = \ell - \min\{D, \ell - 1\}, \dots, \ell - 1 \\ 0 & \text{otherwise} \end{cases}$$

for $\ell, k = 1, 2, \dots, N$. This means that the recursive model is characterized by a Toeplitz matrix \mathbf{A} . It is known that, out of all the size- N Toeplitz matrices over a finite field of q elements, a fraction $1 - 1/q$ (i.e., $1/2$ in our binary case) is non-singular [16]. The considered matrix associated with the recursive model (4) is also lower triangular with equal elements on the main diagonal, since $A_{\ell k} = 0$ for $\ell < k$. Therefore, the determinant can be written as [17]:

$$\det \mathbf{A} = \prod_{\ell=1}^N A_{\ell\ell} = 1.$$

This proves that the inverse matrix exists for this case.

Consider now circulant matrices and restrict to those matrices with coefficients, for $\ell = 1, 2, \dots, N$, of the form:

$$A_{\ell k} = \begin{cases} 1 & \text{for } k = \ell, \ell + 1, \dots, \ell + d - 1 \pmod N \\ 0 & \text{otherwise} \end{cases}$$

where d is such that d consecutive matrix coefficients are equal to 1 and the remaining ones are equal to zero. The mod N operation is needed to perform the circular shift of the rows. The determinant is known if d is a prime [17]:

$$\det \mathbf{A} = \begin{cases} 0 & \text{if } d \mid N \\ d \pmod 2 & \text{otherwise} \end{cases}$$

where the notation $d \mid N$ means that d divides N . Therefore, circulant matrices admit an inverse if d is an odd prime (i.e., $d \neq 2$) and does not divide N .

Note that the circulant case can be shown to be equivalent to the serial model of Fig. 1(b) for $d = 1$ and $\rho_1 = 1/2$. As an example, consider the case $N = 5$, for which the correlation matrix \mathbf{A} is such that the following system of equations can be written:

$$\begin{aligned} X_1 &= Z_1 \\ X_2 \oplus X_3 &= Z_2 \\ X_3 \oplus X_4 &= Z_3 \\ X_4 \oplus X_5 &= Z_4 \\ X_1 \oplus X_5 &= Z_5. \end{aligned}$$

This leads to a model similar to that in Fig. 1(b) with proper rearranging of the cascade of BSCs.

Appendix C. Asymptotic entropy rate of the mixed correlation model

Let us consider the joint entropy of X_{1j}^N and B :

$$H(B, X_{11}^N, \dots, X_{1M}^N) = H(B) + H(X_{11}^N, \dots, X_{1M}^N | B). \quad (25)$$

Given B , the set X_{1j}^N is independent of X_{1k}^N for $j \neq k$ and, therefore,

$$H(X_{11}^N, \dots, X_{1M}^N | B) = \sum_{j=1}^M H(X_{1j}^N | B) = \sum_{j=1}^M \sum_{\ell=1}^N H_b(\rho_{\ell j}).$$

Noting again that B is a uniformly distributed binary random variable with $H(B) = H_b(0.5) = 1$, one obtains:

$$H(B, X_{11}^N, \dots, X_{1M}^N) = 1 + \sum_{j=1}^M \sum_{\ell=1}^N H_b(\rho_{\ell j}).$$

As in (25), it is also possible to write

$$\begin{aligned} 1 + \sum_{j=1}^M \sum_{\ell=1}^N H_b(\rho_{\ell j}) &= H(B, X_{11}^N, \dots, X_{1M}^N) \\ &= H(X_{11}^N, \dots, X_{1M}^N) + H(B | X_{11}^N, \dots, X_{1M}^N) \end{aligned}$$

and, therefore,

$$\begin{aligned} H(X_{11}^N, \dots, X_{1M}^N) &= 1 + \sum_{j=1}^M \sum_{\ell=1}^N H_b(\rho_{\ell j}) \\ &\quad - H(B | X_{11}^N, \dots, X_{1M}^N). \end{aligned} \quad (26)$$

Since by definition of entropy the last term is non negative, the following UB results

$$H(X_{11}^N, \dots, X_{1M}^N) \leq 1 + \sum_{j=1}^M \sum_{\ell=1}^N H_b(\rho_{\ell j}). \quad (27)$$

Moreover, since conditioning reduces entropy [15], it also follows that

$$H(B | X_{11}^N, \dots, X_{1M}^N) \leq H(B | X_{11}) = H_b(\rho_{11}).$$

Using this in (26), one obtains the LB:

$$H(X_{11}^N, \dots, X_{1M}^N) \geq 1 + \sum_{j=1}^M \sum_{\ell=1}^N H_b(\rho_{\ell j}) - H_b(\rho_{11}). \quad (28)$$

Combining (27) and (28), one obtains the limit in (20).

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